JOURNAL
OF THE
WASHINGTON ACADEMY OF SCIENCES

VOL. 22 FEBRUARY 19, 1932 No. 4

MATHEMATICS.—*A comparison of certain symmetrical growth curves.*[1] CHARLES P. WINSOR. (Communicated by RAYMOND PEARL.)

In recent years much has been written on the mathematical representation of growth, both of organisms and of populations. Various equations have been used for the purpose, either selected empirically or derived from more or less rational biological considerations. In particular the equation

$$y = \frac{k}{1 + e^{a - bx}} \qquad (1)$$

derived by Verhulst, and by him called the logistic, has been used extensively. (For full references, see Pearl (5)). As is well known, this curve has asymptotes $y = 0$ and $y = k$, no point of zero slope between the asymptotes, and a point of inflection when $y = k/2$.

It is of course clear that there are many other curves which possess similar properties. The question has been raised (among others, by Bowley (2) and Davies (3) as to why the use of the logistic should be preferable to that of the integrated normal curve.

In general the choice of a mathematical function to represent observed data is influenced by two different considerations. We may have, or think we have, *a priori* knowledge of the mechanics of the phenomenon, from which we may deduce that the data should follow a certain law. More often, in biological work, the underlying causes and their mode of action are so obscure that we are in no position to make sound deductions about laws; we have to infer our law from the observations. We shall, under such conditions, probably be guided

[1] From the Department of Biology of the School of Hygiene and Public Health of the Johns Hopkins University. Received January 2, 1932.

by some such considerations as these: (1) we want a function which is mathematically simple, both in its functional form and in the number of arbitrary constants involved; (2) the function must reproduce the observations with reasonable fidelity; (3) the function must not lead to absurdities on extrapolation. Obviously, these considerations are not sufficient, in the mathematical sense; different workers will interpret them differently, and it may be quite impossible to reconcile differences of opinion which may arise.

In the particular case which interests us here, we seem to be faced with just this kind of problem. There is little understanding or agreement as to the underlying mechanism of growth; there is a considerable body of data on growing individuals and growing populations. In particular, it has been found that many sets of growth data can be fitted, more or less closely, with logistic curves. But clearly this does not exclude the possibility that they might be as well, or better, fitted with some other curve.

It is the primary purpose of the present paper to make a mathematical comparison of certain symmetrical curves, not with observational material, but with each other, with a view to determining how different in form they actually are. Clearly whether it is actually possible to discriminate between functions on the basis of goodness of fit to observations will depend in part on the differences between the curves themselves, and in part on the regularity or irregularity of the observations. It is useful to compare the curves as curves, since we can then form an opinion as to whether observational material is likely to be regular enough to furnish an adequate test of one hypothesis as against another.

The particular curves which will be considered here are the logistic, the integrated normal curve, the arc-tangent curve, and the integrated Pearson Type VII curve.[2] The equations to these curves are:

Logistic:
$$y_L = \frac{k}{1 + e^{a - bx}} \tag{1}$$

Integrated Normal:
$$y_N = \frac{k}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{x - x_0}{\sigma}\right)^2} dx \tag{2}$$

Arc-tangent:
$$y_T = \frac{k}{\pi} \tan^{-1}\left(\frac{x - x_0}{a}\right) + \frac{k}{2} \tag{3}$$

[2] There are of course many other symmetrical growth curves which might with equal reason be used: e.g., $y = K \tan^{-1} e^{ax + b}$

$$\text{Integrated Type VII: } y_P = y_0 \int_{-\infty}^{x} \left(1 + \frac{(x - x_0)^2}{a^2}\right)^{-m} dx \qquad (4)$$

For purposes of comparison, we shall make the upper asymptote unity for all curves; and we shall choose the time origin at the point of inflection. This will leave us one constant in each of the first three curves, and two in the Pearson curve, to be determined. Further, it is convenient to set $\sigma = 1$ in the integrated normal curve, since this enables us to read its ordinates direct from Sheppard's Table. This throws our equations into the form:

$$\text{Logistic:} \qquad y_L = \frac{1}{1 + e^{-bx}} \qquad (1')$$

$$\text{Integrated Normal:} \quad y_N = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}x^2} dx \qquad (2')$$

$$\text{Arc-tangent:} \qquad y_T = \frac{1}{\pi} \tan^{-1}\left(\frac{x}{a}\right) + \tfrac{1}{2} \qquad (3')$$

$$\text{Integrated Type VII: } y_P = \frac{\Gamma(m)}{a\sqrt{\pi}\,\Gamma(m - \frac{1}{2})} \int_{-\infty}^{x} \left(1 + \frac{x^2}{a^2}\right)^{-m} dx \qquad (4')$$

The natural method to follow in fixing the values of the constants of these equations would be the method of least squares; this, however, involves us in difficult integrations, and it is questionable how far the results obtained would be of value in practice.[3] The method of moments is perhaps the next that occurs to one; naturally not the moments of the curves themselves, but of their first derivatives. This method may be followed for comparing equations (1), (2), and (4); but in the case of the arc-tangent curve the second moment of the derivative is infinite.

We may also determine the constants by fixing the points of intersection of the curves. This will be simple in the case of all but equation (4). We shall arbitrarily set the points of intersection at $y = \frac{1}{4}$, $y = \frac{1}{2}$, $y = \frac{3}{4}$. By symmetry, an intersection at $y = (\frac{1}{2} - a)$ determines an intersection at $y = (\frac{1}{2} + a)$.

[3] In practice, we are dealing only with a finite segment of the curve, and in general only with that portion where growth is active; but a least squares solution gives equal weight to differences wherever they occur.

We shall have then two comparisons: (A) equations (1), (2), and (4) compared by the method of moments, and (B) equations (1), (2), and (3) compared by fixing points of intersection at $y = \frac{1}{4}$, $y = \frac{1}{2}$, $y = \frac{3}{4}$.

We obtain the following equations:

Comparison A.

Logistic: $$y_L = \frac{1}{1 + e^{-\pi x/\sqrt{3}}}$$

Integrated Normal: $$y_N = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}x^2}\, dx$$

Integrated Type VII: $$y_P = \frac{2^7}{35\pi\sqrt{7}} \int_{-\infty}^{x} \left(1 + \frac{x^2}{7}\right)^{-4} dx$$

Comparison B.

Logistic: $$y_L = \frac{1}{1 + e^{-1.62880\,x}}$$

Integrated Normal: $$y_N = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}x^2}\, dx$$

Arc-tangent: $$y_T = \frac{1}{\pi} \tan^{-1}\left(\frac{x}{.67449}\right) + \frac{1}{2}$$

Tables 1 and 2 show the ordinates of each of these curves for comparisons A and B respectively; Figures 1 and 2 show the course of the the curves graphically. It will be noted that the Type VII curve gives a much closer fit to the logistic than the normal curve; this, of course, is to be expected, since we have equated one more moment. It will also be observed that in comparison B we have a distinctly closer agreement between the normal curve and the logistic than in Comparison A, over that part of the growth cycle that is of greatest interest to us. We may infer that the method of moments is not the best method for use in this particular case; and that perhaps a still better fit would be obtained by the method of least squares.[4]

The comparison may be made in another way, which may be of interest. Values of $\frac{k - y}{y}$ plotted on semi-log paper give a straight

[4] Subject to the consideration in the previous foot-note.

line if y follows a logistic. If we plot values of $\frac{k-y}{y}$ for other equations, we shall have curves which approach a straight line more or less closely as the curves themselves approach the logistic more or

TABLE 1.—COMPARISON OF INTEGRATED NORMAL, LOGISTIC, AND INTEGRATED TYPE VII CURVES. CONSTANTS DETERMINED BY METHOD OF MOMENTS

x	Integrated Normal	Logistic	Integrated Type VII
0	.5000	.5000	.5000
.2	.5793	.5897	.5871
.4	.6554	.6738	.6695
.6	.7257	.7481	.7433
.8	.7881	.8102	.8060
1.0	.8413	.8598	.8569
1.2	.8849	.8981	.8966
1.4	.9192	.9269	.9266
1.6	.9452	.9479	.9485
1.8	.9641	.9632	.9642
2.0	.9772	.9741	.9752
2.2	.9861	.9818	.9829
2.4	.9918	.9873	.9882
2.6	.9953	.9911	.9919
2.8	.9974	.9938	.9944
3.0	.9987	.9957	.9961

TABLE 2.—COMPARISON OF INTEGRATED NORMAL, LOGISTIC, AND ARC-TANGENT CURVES. POINTS OF INTERSECTION AT $y = \frac{1}{4}$, $y = \frac{1}{2}$, $y = \frac{3}{4}$

x	Integrated Normal	Logistic	Arc-tangent
0	.5000	.5000	.5000
.2	.5793	.5806	.5958
.4	.6554	.6571	.6704
.6	.7257	.7262	.7314
.8	.7881	.7859	.7771
1.0	.8413	.8356	.8111
1.2	.8849	.8756	.8370
1.4	.9192	.9069	.8571
1.6	.9452	.9309	.8730
1.8	.9641	.9491	.8859
2.0	.9772	.9627	.8965
2.2	.9861	9728	.9053
2.4	.9918	.9802	.9128
2.6	.9953	.9856	.9192
2.8	.9974	.9896	.9248
3.0	.9987	.9924	.9296

In these tables ordinates have been tabulated only for positive values of x; for negative values of x, the ordinate $y_{-x} = 1 - y_x$.

less closely. Figure 3 shows the values $\left(\frac{1-y}{y}\right)$ for the curves in comparison B.

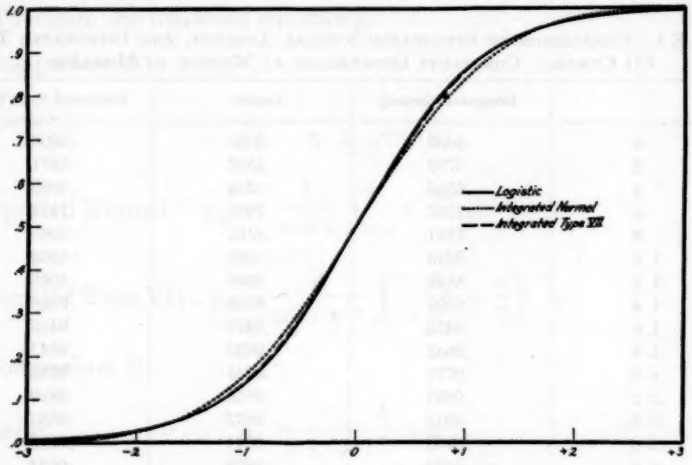


Fig. 1. Logistic, Integrated Normal, and Integrated Type VII curves. Equating moments.

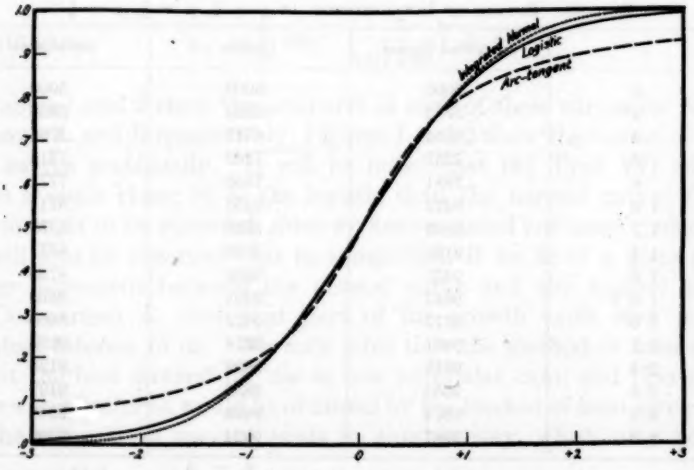


Fig. 2. Logistic, Integrated Normal, and Arc-tangent Curves. Points of Intersection Fixed.

From the discussion above, it appears that the logistic and the integrated normal curve are so nearly similar as to suggest a question as to whether one could discriminate between them on the basis of experimental data. It will be noted that the maximum difference in corresponding ordinates is of the order of 1.5 per cent of the asymptote; and an examination of published observational data will show but little material approaching this order of accuracy. The data of

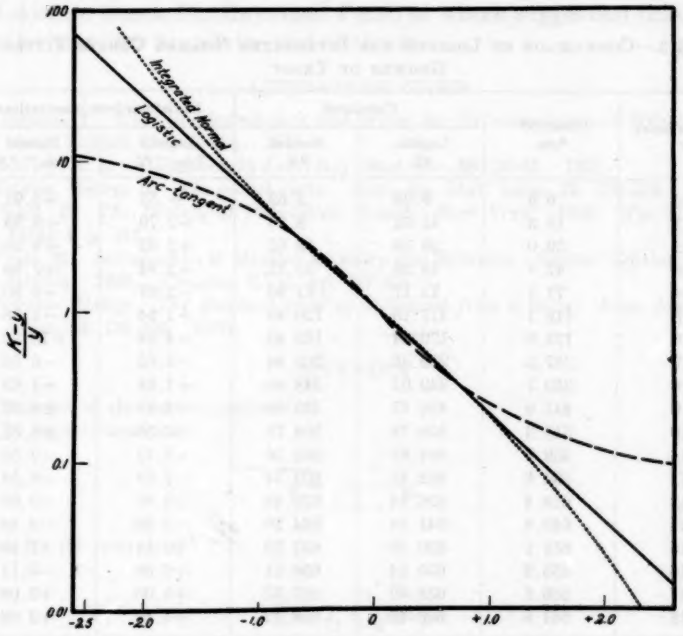


Fig. 3. Values of $\frac{1-y}{y}$ for Logistic, Integrated Normal, and Arc-tangent Curves, on arithlog paper.

Carlson (1) on growth of yeast, which have been worked over by Pearl (4) and Schultz (6), are, however, sufficiently regular and sufficiently extensive to suggest an attempt at a graduation comparison.

Fitting by the method of least squares, we obtain equations as follows:

Logistic: $$y_L = \frac{662.88}{1 + e^{4.27060 - .54823\,x}} \qquad (5)$$

$$\text{Integrated Normal: } y_N = \frac{658.38}{3.0634\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{x-7.7700}{3.0634}\right)^2} dx \qquad (6)$$

Table 3 shows the results of the graduation by these two equations. It will be observed that not merely does the logistic give a better fit, as indicated by the quadratic mean error, but that there is distinctly more tendency to systematic deviations in the case of the normal curve.

TABLE 3.—COMPARISON OF LOGISTIC AND INTEGRATED NORMAL CURVES FITTED TO GROWTH OF YEAST

Time in hours x	Observed[a] y_{obs}	Calculated		Deviations from observations	
		Logistic y_L	Normal y_N	Logistic $y_{obs} - y_L$	Normal $y_{obs} - y_N$
0	9.6	9.05	3.69	+.55	+5.91
1	18.3	15.51	8.95	+2.79	+9.35
2	29.0	26.38	19.62	+2.62	+9.38
3	47.2	44.36	39.31	+2.84	+7.89
4	71.1	73.17	71.90	−2.07	−0.80
5	119.1	117.16	120.48	+1.94	−1.38
6	174.6	179.54	185.40	−4.94	−10.80
7	257.3	259.35	263.94	−2.05	−6.64
8	350.7	349.02	348.88	+1.68	+1.82
9	441.0	436.17	432.03	+4.83	+8.97
10	513.3	509.74	504.78	+3.56	+8.52
11	559.7	564.82	562.26	−5.12	−2.56
12	594.8	602.41	603.34	−7.61	−8.54
13	629.4	626.54	629.48	+2.86	−0.08
14	640.8	641.38	644.29	−0.58	−3.49
15	651.1	650.29	652.39	+0.81	−1.29
16	655.9	655.54	656.01	+0.36	−0.11
17	659.6	658.60	657.52	+1.00	+2.08
18	661.8	660.40	658.12	+1.40	+3.68
Sum of squares of deviations =				194.88	715.20
Quadratic mean error =				3.49	6.69

[a] Data from Carlson's (1) Table VIII.

It seems clear that in this instance, at least, the logistic is essentially a better growth curve than the integrated normal.

It does not appear worth while to attempt a comparison with the arc-tangent curve or with the integrated Type VII curve. The arc-tangent curve will not even approximately fit the data; the Type VII curve will perhaps give a good fit, but only at the cost of an unwarranted amount of effort.

SUMMARY AND CONCLUSION

From a comparison of the logistic equation with those for the arc-tangent curve and the integrated normal curve, it appears that the logistic and normal curves are of closely similar shape, and that both differ widely from the arc-tangent curve. It also appears that the logistic curve describes the growth of a population of yeast cells with distinctly greater accuracy than does the integrated normal curve.

I wish to thank Dr. Raymond Pearl, at whose suggestion this paper was written, for his advice and criticism.

LITERATURE CITED

1. Carlson, T. *Über Geschwindigkeit und Grösse der Hefevermehrung in Würze.* Biochem. Zeitsch **57**: 313–334. 1913.
2. Bowley, A. L. Discussion in Jour. Roy. Stat. Soc. **88**: 76–81. 1925.
3. Davies, George R. *The growth curve.* Jour. Am. Stat. Assoc. **22**: 370–374. 1927.
4. Pearl, R. *The Biology of Population Growth.* New York. 1925. [Pp. 9–10 and Table 4, p. 217.]
5. Pearl, R. *Introduction to Medical Biometry and Statistics.* Second Edition, Philadelphia. 1930. [Chapter XVII, pp. 417–428.]
6. Schultz, Henry. *The standard error of a forecast from a curve.* Jour. Am. Stat. Assoc. **25**: 139–185. 1930.

APPENDIX

1. Moments of the logistic derivative.
The logistic equation

$$y = \frac{1}{1 + e^{-bx}}$$

has for its derivative

$$y' = \frac{be^{-bx}}{(1 + e^{-bx})^2}$$

We require the moments of this derivative curve:

$$\mu_n = \int_{-\infty}^{+\infty} x^n y' \, dx$$

Plainly all odd moments are zero; and also

$$\mu_{2n} = 2b \int_0^{\infty} \frac{x^{2n} e^{-bx} \, dx}{(1 + e^{-bx})^2}$$

Setting $bx = u$, and expanding the denominator we have

$$\mu_{2n} = \frac{2}{b^{2n}} \sum_{k=1}^{\infty} \int_0^{\infty} (-1)^{k+1} u^{2n} k e^{-ku} \, du$$

And changing variables again, setting $ku = v$,

$$\mu_{2n} = \frac{2}{b^{2n}} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^{2n}} \int_0^{\infty} v^{2n} e^{-v} \, dv$$

$$= \frac{2 \cdot (2n)!}{b^{2n}} \sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k^{2n}}$$

But this is a well-known series involving the Bernoullian numbers, so that we have finally

$$\mu_{2n} = \frac{2\left(2^{2n-1} - 1\right) \pi^{2n} B_n}{b^{2n}}$$

$$B_1 = \tfrac{1}{6}, \; B_2 = \tfrac{1}{30}, \text{ etc.}$$

And we have as the moments of the logistic derivative

$$\mu_2 = \frac{\pi^2}{3\, b^2}$$

$$\mu_4 = \frac{7\, \pi^4}{15\, b^4}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = 4.2$$

2. For comparison with the normal curve, we set

$$\mu_2 = 1$$

$$b = \frac{\pi}{\sqrt{3}} = 1.81380$$

3. We have for the values of the constants in the Pearson Type VII curve:

$$m = \frac{5\, \beta_2 - 9}{2\, (\beta_2 - 3)}$$

$$a^2 = \frac{2\, \mu_2\, \beta_2}{\beta_2 - 3}$$

$$y_0 = \frac{N}{a} \frac{\Gamma(m)}{\sqrt{\pi}\, \Gamma\left(m - \tfrac{1}{2}\right)}$$

And setting $N = 1$, $\mu_2 = 1$, $\beta_2 = 4.2$, we have

$$m = 5, \; a^2 = 7, \; y_0 = 0.439990$$

4. To determine constants so that the logistic, integrated normal, and arc-tangent curves shall intersect at $y = \frac{3}{4}$

When $y = \frac{3}{4}$, $x = 0.67449$

for the integrated normal curve.

Logistic:

$$y_L = \frac{1}{1 + e^{-bx}}; \quad -bx = \log_e \left(\frac{1 - y_L}{y_L} \right)$$

$y_L = \frac{3}{4}$, $x = 0.67449$

$b = 1.62880$

Arc-tangent:

$$y_T = \frac{1}{\pi} \tan^{-1} \frac{x}{a} + \frac{1}{2}$$

$$\frac{x}{a} = \tan \left(\pi y_T - \frac{\pi}{2} \right)$$

$y_T = \frac{3}{4}$, $x = 0.67449$

$a = 0.67449$

5. Comparison with Carlson's observations.

Carlson's data are taken from his Table VIII (also given by Pearl (4) and Schultz (6). Schultz has fitted these data by the least squares, but states that a better fit than given by his equation (53) might be obtained by a repetition of the least square procedure. Accordingly the constants in his equation (53) were used as first approximations. The results hardly justified the trouble, as the quadratic mean error was only reduced from 3.50 to 3.49.

For the least square fit to the normal curve, approximate values of the constants were selected as follows:

$$k_0 = 662$$

$$x_{0_0} = 7.8$$

$$\sigma_0 = 3.1$$

The corrections found were:

$$\Delta k = -3.62$$

$$\Delta x_0 = -.030$$

$$\Delta \sigma = -.0366$$

giving final values

$$k = 658.38$$

$$x_0 = 7.770$$

$$\sigma = 3.0634$$

A comparison of the retained and neglected terms for 0, 9, and 18 hours indicated that the neglected (second order) terms were actually small in comparison with the retained terms.

GEOLOGY.—*Faunal zones in the Miocene Choctawhatchee formation of Florida.*[1] WENDELL C. MANSFIELD, U. S. Geological Survey, and GERALD M. PONTON, Florida Geological Survey.

In March, 1930, the writers discovered fossiliferous outcrops of Miocene beds in the valley of Alaqua Creek, Walton County, Florida, farther south than fossils had previously been reported from this region.[2] Fossils from these beds, when studied by the senior author, showed that they belong to the Choctawhatchee formation. They not only give more substantial evidence of the close faunal relationship of the Shoal River formation to the succeeding Choctawhatchee formation but also reveal the sequence of the zones of the Choctawhatchee formation and the position in the formation of the typical deposits at Red Bay.

The map (Figure 1) shows the type localities of the Shoal River and Choctawhatchee formations and the places at which fossils have been collected near Alaqua Creek. The numbers on the map are the serial numbers recorded in the station book of the U. S. Geological Survey kept at the National Museum. The boundary between the Shoal River formation and the Choctawhatchee formation as drawn is entirely conjectural. The newly discovered localities are stations 12044–48, 12060, and 12527. The following are the explanations of the station numbers:

3742. Shell Bluff. Type locality of the Shoal River formation.
3747. Parker place. Shoal River formation.
4975, 7152. Red Bay. Choctawhatchee formation (*Ecphora* zone and upper part of *Arca* zone).
5618. Langley's old farm. Shoal River formation.
9959. One-fourth mile west of Pleasant Ridge Church. Shoal River formation.
10612. Chester Spence's farm. Provisionally placed in the Shoal River formation.

[1] Published by permission of the Director of the U. S. Geological Survey and of the State Geologist of Florida. Received January 12, 1932.

[2] Station 3747, reported by Gardner (U. S. Geol. Survey Prof. Paper 142) as 8 miles south of Lake DeFuniak, is 8 miles nearly due west of DeFuniak Springs, in SW. ¼ sec. 34, T. 3 N., R. 20 W.—an error of the clerk in copying the station record. The shells were found at a depth of 30 feet in a well dug for water.

12044. Bell farm, upper locality. Choctawhatchee formation (*Arca* zone).
12045. Bell farm, lower locality. Choctawhatchee formation (*Arca* zone).
12046. Vaughan Creek, upper locality. Choctawhatchee formation (*Arca* zone).
12047. Vaughan Creek, lower locality. Choctawhatchee formation (*Arca zone*).
12048. Permenter's old place. Choctawhatchee formation (*Ecphora* zone).
12060. Frazier's old farm. Choctawhatchee formation (*Yoldia* zone).
12527. Alice Creek. Choctawhatchee formation (upper part of *Arca* zone).

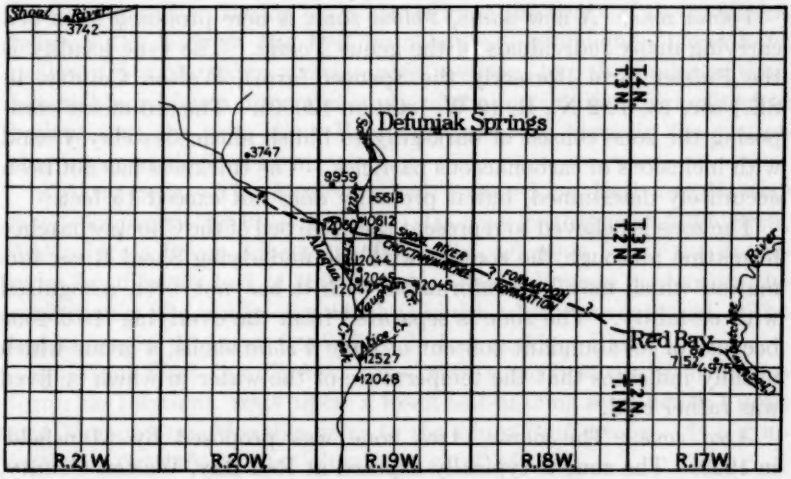


Fig. 1. Fossiliferous localities in Walton County, Florida.

The most southerly locality in the valley of Alaqua Creek at which fossils had previously been collected is the Chester Spence farm in the NE.¼ sec. 17, T. 2 N., R. 19 W. (U. S. G. S. station 10612). The fauna at this place appears to be transitional from that of the Shoal River formation to that of the Choctawhatchee formation. This fauna is provisionally left in the Shoal River formation, where it was placed by Gardner; but it may eventually be placed in the basal part of the Choctawhatchee formation.

The following generalized section shows the divisions that are now recognized in the Choctawhatchee formation.

GENERALIZED SECTION OF THE CHOCTAWHATCHEE FORMATION

	Feet
5. *Cancellaria* zone. Fine to coarse clayey fossiliferous sand	25–30
4. "Aluminous clay." Grayish unfossiliferous clay	25
3. *Ecphora* zone. Sandy fossiliferous clay	15–25
2. *Arca* zone. Gray sandy fossiliferous marl	55
1. *Yoldia* zone. Dark-gray to bluish micaceous and carbonaceous clayey fossiliferous sand	15

The *Yoldia* zone, which is here recognized for the first time, and the *Arca* zone are now regarded as representing the upper part of the middle Miocene; the *Ecphora* zone, the "aluminous clay," and the *Cancellaria* zone are referred to the upper Miocene.

Yoldia zone.—A new name, *Yoldia* zone, is here proposed for a bed carrying many individuals of the genus *Yoldia*. The type locality is the Frazier farm (formerly the Spencer farm), Walton County, in SE.¼ sec. 18, T. 2 N., R. 19 W. (station 12060). The sediments composing the zone consist of dark-gray to bluish micaceous clayey sand with inclusions of carbonaceous particles. The thickness has not been accurately determined, but it probably does not exceed 15 feet.

The zone is believed to represent the basal bed of the Choctawhatchee formation although the contact with the underlying Shoal River formation which may be conformable with it has not been recognized with certainty. The zone is separated from the overlying *Arca* zone because of its abundant content of large *Yoldia* shells, a genus which usually indicates that the temperature of the water in which it lived was rather cold.

Arca zone.—The name *Arca* zone was proposed by Mansfield[3] in 1929. The zone is typically exposed at Red Bay, Walton County, where it forms the lowermost fossiliferous bed, about 21 feet thick, in the exposure (stations 4975, 7152). A nearly unfossiliferous upper bed of clay at this locality, which was formerly included in the *Arca* zone, is now placed in the *Ecphora* zone.

The *Arca* zone consists mainly of very fossiliferous gray sandy marl having an estimated total thickness of about 55 feet. It probably rests conformably upon the *Yoldia* zone. The upper limit of the *Arca* zone is provisionally placed at the contact of the marl with an overlying plastic clay bed which, in the section at Red Bay, carries no determinable fossils. The shells in the marl are worn and broken.

[3] W. C. MANSFIELD in C. W. COOKE and STUART MOSSOM, *Geology of Florida*, Florida Geol. Survey Ann. Rept. **20**: 140. 1929; and W. C. MANSFIELD, *Miocene gastropods and scaphopods of the Choctawhatchee formation of Florida*, Florida Geol. Survey Bull. **3**: 15. 1930.

The absence of fossils from the clay and the lithologic difference between the marl and the clay suggest an unconformity between the two beds, but this relationship has not been fully established.

The *Arca* zone was observed in the Alaqua Creek Valley at the head of small branches flowing into Sconiers Mill Creek, on the Bell farm, in the NE.¼ sec. 29, T. 2 N., R. 19 W. (stations 12044–45); on Vaughan Creek, in secs. 27 and 28, T. 2 N., R. 19 W. (stations 12046–47); and at Alice Creek, in the SE.¼ sec. 8, T. 1 N., R. 19 W. (station 12527).

The beds exposed at the Bell farm and along Vaughan Creek are believed to carry the earliest fauna of the *Arca* zone, whereas the lower fossiliferous bed at Red Bay carries the latest fauna of this zone. The senior author, basing his evidence upon the study of the mollusks, believes the beds at the Bell farm and along Vaughan Creek have nearly if not the same stratigraphic position, but the junior author, basing his evidence upon the study of the foraminifera, is inclined to believe that the beds along Vaughan Creek are lower in the section than those at the Bell farm.

Ecphora zone.—The *Ecphora* "bed," named by Dall and Harris,[4] is now known as the *Ecphora* zone.[5] Its type locality is at Alum Bluff, Apalachicola River, Liberty County, Fla., where it forms the uppermost fossiliferous bed of the section. The sediments composing the zone consist of a sandy clay which is bluish where unweathered. The bed ranges in thickness from 15 to 25 feet at Alum Bluff.

At Alum Bluff the *Ecphora* zone, with somewhat doubtful unconformable relations, rests upon a fossil leaf-bearing sand which Cooke and Mossom[6] questionably refer to the Alum Bluff group. It is conformably overlain by the "aluminous clay" of Dall.

An exposure in the east bank of Alaqua Creek on Permenter's old place, in sec. 17, T. 1 N., R. 19 W. (station 12048) apparently represents the *Ecphora* zone. At Red Bay the upper poorly fossiliferous plastic clay bed, about 27 feet thick, is placed in this zone.

The "aluminous clay."—The "aluminous clay," a name applied by Dall[7] to a 25-foot bed of grayish clay overlying the *Ecphora* zone at Alum Bluff, Liberty County, Fla., has not been recognized in the Alaqua Creek Valley.

[4] W. H. Dall and G. D. Harris, *The Neocene of North America*, U. S. Geol. Survey Bull. **84**: 123–124. 1892.

[5] W. C. Mansfield in *Geology of Florida*, p. 140, 1929.

[6] C. W. Cooke and Stuart Mossom, *Geology of Florida*, Florida Geol. Survey Ann. Rept. **20**: 108. 1929.

[7] W. H. Dall and Joseph Stanley-Brown, *Cenozoic geology along the Apalachicola River*, Geol. Soc. Am. Bull. **5**: 168–169. 1894.

Cancellaria zone.—The name *Cancellaria* zone was proposed by Mansfield[8] to include beds that carry the latest Miocene fauna. This zone is typically exposed in the highest fossiliferous beds along Harveys Creek, in the SW.¼ sec. 9, T. 1 S., R. 3 W., Leon County, Fla. The *Cancellaria* zone is composed of fine to coarse grained clayey sand replete with fossils, having an estimated total thickness of 25 to 30 feet. It has not been recognized in the Alaqua Creek Valley.

BOTANY.—*New species of slime molds.*[1] G. W. MARTIN, State University of Iowa. (Communicated by WILLIAM R. MAXON.)

In the present paper six species of Myxomycetes are described as new,—one from Colombia, one from Ontario, and four from the western United States. The descriptions of the two species of *Cribraria* are based in part upon the monographic study of the genus made by Miss Eunice Lovejoy and filed, as a thesis, in the library of the State University of Iowa. The type specimen of the Colombian species and portions of the type specimens of the others are deposited in the United States National Herbarium.

Badhamia cinerascens Martin, sp. nov.

Peridia sessilia, globosa vel leniter depressa, 0.7–1.5 mm. lata, agglomerata; tunica tenuis, fragilis, cinerea, rete calcifero obducta; capillitium album; sporae liberae, fuscae, valide echinulatae, 12–15 μ diam.

Sporangia globose or flattened, sessile or occasionally borne on a pallid, membranous stipe, 0.7–1.5 mm. in diameter, densely aggregated and more or less superimposed, on a pallid membranous hypothallus; peridium thin, fragile, ashy, covered by a dense network of calcareous thickenings; capillitium abundant, white, badhamioid under lens, but under the microscope exhibiting numerous threadlike tubules; spores intensely black in mass, spherical, nonadherent, deep blackish brown by transmitted light, densely and strongly spinulose, 12–15 μ, averaging 13.5 μ, 2 μ representing the spiny margin.

COLOMBIA: On tree trunk, La Sierra, Antioquia, alt. 2,000 m., March 8, 1931, *W. A. Archer* 1662 (type, U. S. Nat. Herb.).

Close to *B. macrocarpa* and, like that and related species, with a more or less physaroid capillitium, but distinguished by its ashy color, the heaped sporangia, and the extremely dark, coarsely and densely spiny spores. In appearance not unlike some specimens of *Physarum cinereum*, but the capillitium distinctly more badhamioid than physaroid, and the spores much larger, darker, and rougher.

[8] W. C. MANSFIELD in *Geology of Florida*, p. 140. 1929.

[1] Received December 30, 1931.

Amaurochaete ferruginea Macbr. & Martin, sp. nov.

Aethalium pulvinatum, longitudine 7 cm.; peridium fugaceum; capillitium ferrugineum; sporae ferrugineae, minute verrucosae, 7.5–9 μ diam.

Aethalium pulvinate, flat, up to 7 cm. in length and 4 cm. in width; peridium fugaceous; hypothallus shining, silvery, extending somewhat beyond the margin of the aethalium; definite columellae lacking, but capillitium branching from numerous rigid irregular branches arising from the hypothallus and soon dissipated into subordinate branches; threads dark brown, bearing numerous lighter brown irregular membranous expansions; spores cinnamon-drab to benzo-brown (Ridgway) in mass, pale reddish brown by transmitted light, minutely warted, 7.5–9 μ.

CALIFORNIA: On charred coniferous wood, Yosemite Park, Aug. 31, 1905, *T. H. Macbride* (type, in herb. State Univ. Ia., no. 1438). OREGON: On decorticated coniferous wood, S. U. I. 1439.

The structure of the capillitium is very similar to that of *A. fuliginosa*, from which species this differs in the brownish color of the capillitium and in the small, pale, relatively smooth, ferruginous spores, the two characters together giving the fructification a ferruginous cast in marked contrast to the black of the other species of the genus.

Amaurochaete trechispora Macbr. & Martin, sp. nov.

Aethalium pulvinatum, longitudine 7 cm.; peridium obscurum, nitens, fugaceum, tuberculatum; capillitium nigrum; sporae atroviolaceae, reticulatae, 13–15 μ diam.

Aethalium pulvinate, flat, up to 7 cm. in length; cortex dark, shining, evanescent, faintly tuberculate as though suggesting the tips of component sporangia; hypothallus broadly expanded, persistent, extending well beyond the borders of the aethalium, silvery, with yellowish stains, and amber globules representing remnants of the presumably yellow plasmodium; capillitium black, irregular, composed of numerous stout columella-like bases, these soon becoming dissipated into numerous freely anastomosing branches; peripheral nets lacking; spores purplish black in mass, lilaceous brown by transmitted light, globose, ornamented with a pronounced reticulation formed of wing-like ridges, the meshes coarse and often unequal, 13–15 μ in diameter, 10–12 μ representing the diameter of the body of the spore, the balance the ridges of the reticulum.

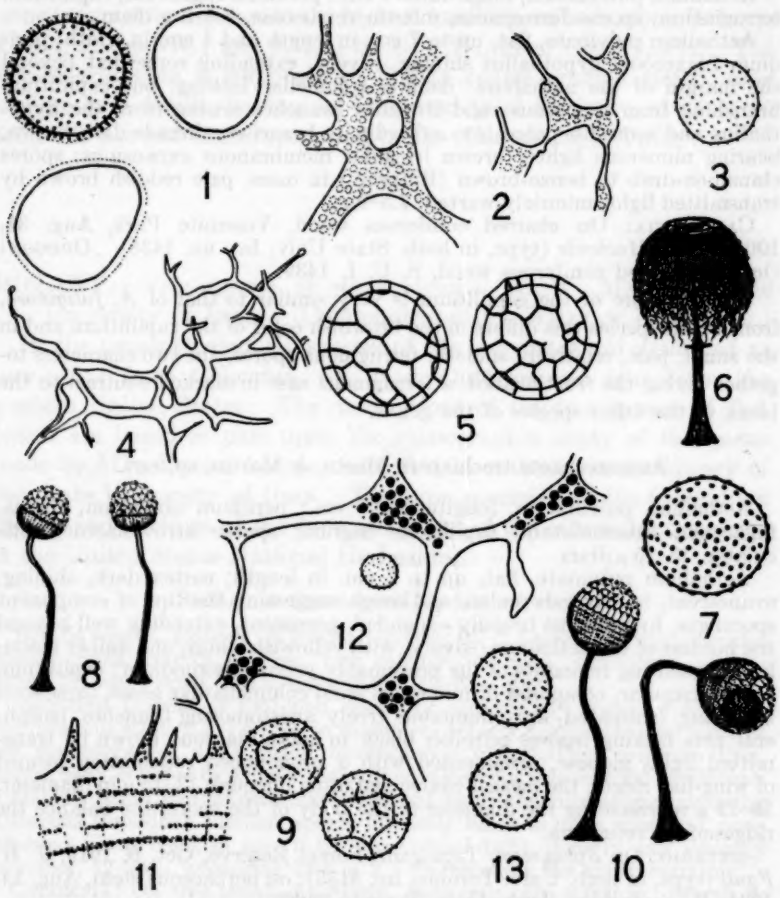
ONTARIO: On *Sphagnum*, Temagami Forest Reserve, Oct. 6, 1919, *J. H. Faull* (type, in herb. Univ. Toronto, no. 5135); on herbaceous stem, Aug. 14, 1931, *H. S. Jackson* (herb. Univ. Toronto 2460).

A well-marked species, related to *A. fuliginosa* but separated by its remarkable and striking spores. Based on a collection well described by Miss Currie[2] and by her doubtfully referred to *Stemonitis fusca* Roth var. *trechispora* Torrend. Aside from the reference to the strongly reticulated spores and the occurrence on *Sphagnum*, there is nothing in Torrend's brief description of his variety[3] to suggest the present species, nor can it be the form

[2] Trans. Royal Can. Inst. 1919: 296.

[3] Fl. Myx. 141. 1909.

described and illustrated by Jahn[4] as *Stemonitis trechispora* Torr. It is clearly an *Amaurochaete*.



Figs. 1–2, *Badhamia cinerascens*, sp. nov. 1, Three spores, two in outline to show common variations in shape, × 1500. 2, Portions of capillitium, badhamioid at left, somewhat physaroid at right, × 164. Fig. 3, *Amaurochaete ferruginea*, sp. nov., spore, × 1500. Figs. 4–5, *Amaurochaete trechispora*, sp. nov. 4, Portion of capillitium, × 164. 5, Two spores, × 1500. Figs. 6–7, *Enerthenema melanospermum*, sp. nov. 6, Fructification, × 20. 7, Spore, × 1500. Figs. 8–9, *Cribraria dictyospora*, sp. nov. 8, Two sporangia, × 20. 9, Two spores, × 1500. Figs. 10–13, *Cribraria atrofusca*, sp. nov. 10, Two sporangia, × 20. 11, Margin of calyculus, showing concentric lines of granules, × 164. 12, Portion of net and spore, × 682. 13, Two spores, × 1500.

[4] Ber. Deutsch. Bot. Ges. **41**: 394. 1923.

Enerthenema melanospermum Macbr. & Martin, sp. nov.

Peridia stipitata, sphaeroidea vel ovata, nigra, 0.8–1 mm. lata, 2 mm. alta; stipes niger, crassus, sub apicem attenuatus; sporae atro-olivaceae, crassiter verrucosae, 12–14 μ diam.

Sporangia intense black, gregarious in small clusters of three to a dozen, these in larger aggregations, globose or oval, stalked, 0.8 to 1 mm. in diameter, total height 2 mm. or more; stipe black, shining, rather stout, attenuate upward and continued as a slender unbranched columella capped with a very large, shining, infundibuliform terminal disk, up to 0.5 mm. in diameter; capillitium dense, black, rather freely branched, arising from terminal disk and with ends free; spores free, dark olivaceous, coarsely warted, 12–14 μ.

OREGON: Three Sisters Mountain, *T. H. Macbride* (type, in herb. State Univ. Ia., no. 1437).

Obviously close to *E. papillatum*, but clearly distinct by reason of its large size, the intense and permanent black color, the very large apical disk, and the large, dark, very rough spores.

Cribraria dictyospora Martin & Lovejoy, sp. nov.

Peridia globosa, erecta, atrofusca, 0.4–0.8 mm. lata; calyculus partem tertiam peridii occupans, margine denticulatus; nodi crassi, atri; sporae violaceae, coacervatae, ochraceo-brunneae, minute verrucosae et ampliter reticulatae, 8–8.8 μ diam.

Sporangia gregarious, dark purplish brown, erect or slightly nodding, globose, 0.4–0.8 mm. in diameter, total height 1–2 mm.; calyculus occupying about one-third of the spore case, marked with irregular, dark, granular rays, the margin toothed; net rather fine-meshed, the connecting threads narrow, the nodes flat and angular, not greatly thickened, densely filled with large, dark granules, making them appear black, free ends abundant, often branched, arising both from nodes and from connecting threads; stipe slender, two or three times the diameter of the sporangium, furrowed, light at the apex, otherwise dark; spores ochraceous brown in mass, clear violet by transmitted light, globose or somewhat angular, minutely warted, and covered with a coarse and often imperfect reticulum of three to five meshes to the hemisphere, 8–8.8 μ, averaging 8.5 μ.

OREGON: On dead wood, other collection data lacking (type, in herb. State Univ. Ia., no. 1435). A different collection, also from Oregon, herb. S. U. I. 1436.

No. 1435, designated as the type, is the more ample collection. The sporangia are slightly smaller than those of no. 1436, the stalks relatively longer, the spores a trifle larger (averaging 8.5 μ as compared with 8.3 μ) and the reticulations slightly less conspicuous. They are, clearly, slightly different aspects of the same species.

The most striking characteristic of this species is the reticulation of the spores, otherwise unknown in the genus. The nodes are similar to those of *C. macrocarpa*, but the granules with which they are filled are larger and much darker. The margin of the calyculus, with its granular rays, suggests that of *C. piriformis*.

Cribraria atrofusca Martin & Lovejoy, sp. nov.

Peridia globosa vel obovata, atrofusca, nitentia, 0.4–0.6 mm. lata, erecta; calyculus partem dimidiam peridii occupans, intus concentricus, margine denticulis longis praeditus; nodi dilati, granulati; sporae atrofuscae, tenuiter verrucosae, 7.5–8.5 μ diam.

Sporangia loosely gregarious, dark purplish brown to nearly black, shining, iridescent, globose or somewhat obovate or occasionally pyriform, usually erect, 0.4–0.6 mm. in diameter, total height 1–2 mm. or more; calyculus occupying nearly or quite one-half of spore case, marked by slender granular ribs radiating from the stipe and by broken concentric granular thickenings deposited on the inside, the concentric character being visible without under the lens in brilliant light, the margin with very fine teeth and long, slender toothlike projections, these bearing the net and similar to its nodes; net regular, with broad connecting threads, the nodes expanded, granular, dark brown, with a few free ends arising from both nodes and threads, the silvery peridium tending to persist; hypothallus small; stipe dark brown or nearly black, slender, furrowed, 0.6–1.8 mm. long; spores dark reddish brown in mass, grayish brown by transmitted light, finely verrucose, 7.5–8.1 μ, averaging 7.9 μ.

COLORADO: On coniferous wood, *T. H. Macbride* (type, in herb. State Univ. Ia., no. 1103); on coniferous wood, *E. Bethel*, S. U. I. 1449; on coniferous wood, 1909, *E. Bethel*, S. U. I. 1450; on coniferous wood (locality not given, but probably Colorado), S. U. I. 1451.

A notable species. The dark, glistening sporangium, the dark spores, and the granular concentric rings within the calyculus are diagnostic. The toothlike projections which bear the net are longer and more slender than in any other species, but their structure suggests that they are to be regarded as elements of the net rather than of the calyculus. The peridium tends to be more persistent than in most cribrarias and in its shining silvery character suggests *Lamproderma arcyrionema*. The spores are much the color of some of the more ferruginous species of *Stemonitis*.

BOTANY.—*The distribution of* Dictyostelium *and other slime molds in soil.*[1] KENNETH B. RAPER and CHARLES THOM, Bureau of Chemistry and Soils.

In our previous paper (7) it was reported that amoeboid organisms were present in abundance in all samples of soil examined. From certain of these samples plasmodia developed indicating that many of these amoeboids represented only a stage in the life cycle of the Myxomycetes. The fruiting bodies of *Dictyostelium* belonging to the Acrasieae, a group of organisms related to the Myxomycetes but which do not have a flagellate stage, and which fruit through a mass movement of amoebae instead of the formation of true plasmodia, also developed in cultures from some samples.

[1] Received January 8, 1932.

Following this we encountered these organisms so frequently in our cultures and from such varied sources as to indicate that they were more abundant and widely distributed than previous reports showed. Since the genera of the Acrasieae pass the whole of their vegetative period as amoeboids and merely form aggregates in their fruiting phase, their identification even to the group depends upon finding culture methods which enable them to complete their life cycle. Thus the possibility was apparent that *Dictyostelium* and the closely related genus *Polysphondylium* when present might easily be overlooked.

It is equally apparent, that special culture procedures must be used in isolating and identifying these organisms. The methods employed in this investigation have been rather simple. The sample of soil or decaying vegetation was thoroughly ground in a clean mortar and diluted with approximately ten volumes of sterile water. The resulting suspension was then streaked upon freshly solidified mannite agar (Ashby's formula) plates, about four to five drops being used to each plate. Incubation was for two to three weeks at room temperature or in an incubator at 16–18°C. The necessary incubation period at the lower temperature is slightly longer but certain fungi are held in check which at the higher temperature tend to overrun the plates.

The fruiting bodies of *Dictyostelium* when present are easily recognized under low magnifications. They appear very much like fruiting structures of some mucors, but the absence of any mycelium leading away from the base readily identifies them as belonging to the Acrasieae. The stalk is considerably swollen at the base and tapers considerably toward the top. *Polysphondylium*, with its verticillately branched sporophore simulates certain mycelial fungi in appearance but can likewise be readily recognized by the absence of a mycelium. With a compound microscope either is at once identifiable by the peculiar and characteristic structure of the stalk, which is formed of polygonal cells, arising from amoebae during fruiting, piled one on top of the other.

Dictyostelium grows slowly and poorly on mannite agar, hence it is best to transfer as soon as possible to more favorable media. Mannite agar is used in isolating these organisms, not because it is a particularly favorable medium for them, but because it is quite unfavorable to most fungi and some bacteria, which, if a stronger medium were used, would overrun the plates in a very few days to such an extent that fruiting bodies of *Dictyostelium* would not be able to develop or not found if they did develop. Fair growth has been obtained upon hay

infusion, carrot or a weak horse dung decoction agar. Moist hay has been used with some success. But we have found a medium consisting of mannite agar plus rat dung[2] to be most satisfactory. Excellent growth is obtained upon this medium and the organisms are easily kept in culture by transferring every four to five weeks. The cultures tend to die off after prolonged artificial cultivation.

Polysphondylium grows better than *Dictyostelium* upon mannite agar. Good growth is obtained upon hay and mannite-dung agars. A weak horse dung decoction agar also gives excellent growth.

Harper used a weak dung decoction in his studies on *Dictyostelium* (2) and *Polysphondylium* (3). Olive (5) employed both dung and peptone agar in his work on the Acrasieae. Skupienski (6) grew *Dictyostelium mucoroides* on weak hay infusion agar.

Our interest in these genera was increased when it became evident that their amoeboid phase appeared in samples of soil from widely different areas. A survey of a considerable series of soil samples was therefore undertaken. This paper is intended to give a brief review of this survey.

During this study *Dictyostelium* has been identified in plates from fifteen samples of various kinds collected in Washington, D. C., and in nearby Maryland and Virginia. Included in these were samples of decaying vegetation such as leaves of curly dock, leaves and stems of *Erigeron*, stems of ragweed, dead blue grass and feather grass, oak leaves from the forest, leaf mould underlying oak and maple leaves, and dead grass floating in a pond. It has been obtained a number of times from soils underlying grass, both at the surface and at a depth of three inches, and from forest soil underlying leaf mould. It has also been isolated from stagnant water.

Samples for this study were collected in some of the eastern states during September, 1931. Both *Dictyostelium* and *Polysphondylium* were identified in, and isolated from a considerable number of these samples. Table 1 best shows the diverse types of samples studied, the substrata upon which the two genera were found and the place of collection.

In the course of other studies, samples of cultivated field soils from a number of western states were collected by this laboratory during the fall of 1930. They were air dried when collected and remained in this condition in the laboratory. Almost a year later, in September and October, 1931, some of these soils were plated on mannite agar in

[2] Mannite agar plates to which sterile rat dung is added before the agar solidifies.

the usual way, incubated at 18–20°C. for three weeks, and studied for presence of these organisms. *Dictyostelium* appeared in many of the plates. The results obtained are given according to states. *Dictyostelium* developed in four of ten samples from Utah, in three of seven from Nebraska, in four of ten from North Dakota, in two of six from Colorado; while it was not found in the three samples from Iowa. All of these western soils were definitely alkaline, some even as much as pH 8.40, whereas the eastern soils studied were all more or less acid.

The finding of *Dictyostelium* in so many of these soils was especially interesting as showing that members of this genus are not uncommon in cultivated fields in the plains area but normally constitute a part of the micropopulation. It indicates that members of the genus are

TABLE 1

No.	Genus Found	Nature of Sample	Place of collection
T4	Dictyostelium Polysphondylium	Rotting wood	Staunton, Va.
T5	Polysphondylium	Forest soil and humus	Clifton Forge, Va.
T10	Dictyostelium	Forest soil	Johnson City, Tenn.
T20	Dictyostelium	Phosphatic soil, field	Lexington, Ky.
T28	Dictyostelium	Decomposing leaves, hardwood forest	North Vernon, Ind.
T33	Dictyostelium	Forest leaves	Scottsburg, Ind.
T37	Polysphondylium	Decaying holly leaves	Leonardtown, Md.
T39	Polysphondylium	Pine needles	Leonardtown, Md.
T41	Polysphondylium	Soil underlying pine needles	Leonardtown, Md.
176B	Dictyostelium	Leaf mould layer, hardwood forest	Plainsville, N. Y.

widely distributed as soil organisms, presenting much the same picture in these soils as in soils collected in and around Washington, D. C. It showed clearly that *Dictyostelium* can retain its viability in spite of prolonged dessication, either as spores or encysted myxoamoebae, microcysts.

Until recently these organisms were reported and considered primarily as coprophilous species, which might occasionally be found growing upon decaying vegetation. That view is no longer tenable in view of the studies made by Krzemieniewski (4) who found *Dictyostelium mucoroides* in almost all soils examined, and *Polysphondylium violaceum*, though rarely, in uncultivated soils. Similarly, Harper (3) isolated *Polysphondylium violaceum* from soil in the parks of New York City. This paper extends these American observations to apply to wide areas.

In our studies, species have not been accurately determined, but all cultures of *Dictyostelium* isolated, with two exceptions, appear to belong to the single species *D. mucoroides*. *Polysphondylium violaceum* is the only species of that genus isolated.

CONCLUSION

Attention is therefore called to species of these genera as constituting a part of the normal microflora of soils and decaying vegetation. They should be taken into account in any soil population studies. Although the forms thus far recognized do not account for all of the amoeboid forms found in soil and decaying vegetation of various kinds, they undoubtedly, as shown here, do account for many. Improved culture methods may enable us to identify still more of the "soil amoebae" as belonging in the slime mold group.

LITERATURE CITED

1. Brefeld, O. Polysphondylium violaceum *and* Dictyostelium mucoroides. Schimmelpilze **6**: 1–34. 1884.
2. Harper, R. A. *Morphogenesis in* Dictyostelium. Bull. Torrey Bot. Club. **53**: 229–268. 1926.
3. Harper, R. A. *Morphogenesis in* Polysphondylium. Bull. Torrey Bot. Club **56**: 227–258. 1929.
4. Krzemieniewski, Helena S. *Z Mikroflory gleby w Polsce. (Contribution a la microflore du sol en Pologne)* Acta. soc. Bot. Poloniae **4**: 141–144. 1927.
5. Olive, E. W. *Monograph of the Acrasieae.* Proc. Boston Soc. Nat. History, **30**: 451–513. 1901.
6. Skupienski, F. X. *Sur la sexualite chez une espèce de Myxomycete Acrasies* Dictyostelium mucoroides. Comptes Rendu **167**: 960–962. 1918.
7. Thom, C., and Raper, Kenneth B. *Myxamoebae in soil and decomposing crop residues.* This JOURNAL **20**: 362–370. 1930.

SCIENTIFIC NOTES AND NEWS

DR. T. W. STANTON has been appointed Chief Geologist of the U. S. Geological Survey, effective February 1.

COL. C. H. BIRDSEYE has been reinstated in the U. S. Geological Survey as a principal engineer and entered upon that duty February 1. He will serve for the present as consultant to the engineering branches and the Director.

Announcement has been received from the permanent committee of the International Congress of Zoology that in accordance with the resolution voted by the 11th Congress at Padua in September, 1930, and after obtaining consent from the Portuguese authorities, the 12th International Congress will be held at Lisbon during the summer of 1935, under the presidency of DR. ARTHUR R. JORGE, Professor in the University of Lisbon and Director of the Musée Bocage.

DR. F. H. H. ROBERTS, JR., archaeologist in the Bureau of American Ethnology, has been detailed to the Carnegie Institution of Washington to serve as consulting archaeologist in connection with the Carnegie's excavations at Chichen Itza. DOCTOR ROBERTS sailed from New York on February 2, and will not return until after the first of March.